Semi-annual Workshop in Dynamical Systems and Related Topics
Pennsylvania State University, November 3–6, 2022

Organizers: Jairo Bochi and Svetlana Katok

Schedule of Talks

Thursday, November 3, 2022		**Friday, November 4, 2022, Pesin session**		**Saturday, November 5, 2022, Zhenqi Wang session**		**Sunday, November 6, 2022**	
		9:00 - 9:50	Hasselblatt	9:00 - 9:50	Kucherenko	9:00 - 9:50	Khanin
		9:50 - 10:10	*Coffee break Poster session*	9:50 - 10:10	*Coffee break Poster session*	10:00 - 10:30	*Coffee break Poster session*
		10:10 - 11:00	Climenhaga	10:10 - 11:00	Pène	10:30 - 10:55	114MB: Ruan 113MB: Wood
						11:00 - 11:25	114MB: Butt 113MB: Corso
		11:10 - 12:00	Jakobson	11:10 - 12:00	Erchenko	11:30 - 11:55	114MB: Oregon Reyes 113MB: Paradela
						12:00 - 12:25	114MB: Osterman 113MB: Bhullar
		12:00 - 1:30	*Lunch break*	12:00 - 1:30	*Lunch break*	12:30 - 1:20	Shub
1:00 - 1:50	*Registration*	1:30 - 2:20	Dolgopyat	1:30 - 2:20	Wang		
1:50 - 2:00	*Opening remarks*	2:30 - 3:20	Ledrappier	2:20 - 2:35	Junior Brin Prize Ceremony		
2:00 - 2:50	Smerkin	3:20 - 3:40	*Coffee break Poster session*	2:40 - 3:00	*Coffee break*		
3:00 - 3:30	*Departmental Tea*	3:40 - 4:30	Luzzatto	3:00 - 3:50	Talk by the winner		
3:30 - 4:30	Colloquium: Szász	4:40 - 5:30	Burns	4:00 - 4:50			
		6:30 - 10:00	*Banquet at at Hilton Garden Inn, State College*				

Titles and Abstracts

Keith Burns (Northwestern University)
Title: *Uniqueness of the measure of maximal entropy for geodesic flows on surfaces with caps.*
Abstract: The class of surfaces in this talk was introduced in the 1980s by Donnay in order to exhibit a smooth Riemannian metric on the two sphere with ergodic geodesic flow. The geodesic flows for these surfaces have unique (and therefore ergodic) measures of maximal entropy. The proof uses Climenhaga and Thompson's extension of the approach pioneered by Bowen and Franco. This is joint work with Todd Fisher and Rachel McEnroe.

Jasmine Bhullar (Univerity of Houston)
Title: *$\overline{d}$-continuity for Countable-State Markov Shifts.*
Abstract: The concept of $\overline{d}$-distance on the space of invariant measures on a shift space was introduced by Ornstein to study the isomorphism problem for Bernoulli shifts. For mixing subshifts of finite type, Coelho and Quas showed that the map that sends a Hölder continuous potential ϕ to its equilibrium state μ_ϕ is $\overline{d}$-continuous. In this talk, we extend this result to the setting of Countable State Markov Shifts. We show that the map that sends a strongly positive recurrent potential to its normalization is continuous, and then use the machinery of g-functions to prove the extension.

Karen Butt (University of Michigan, Ann Arbor)
Title: *Quantitative marked length spectrum rigidity.*
Abstract: The marked length spectrum of a closed Riemannian manifold of negative curvature is a function on the free homotopy classes of closed curves which assigns to each class the length of its unique geodesic representative. It is known in certain cases that the marked length spectrum determines the metric up to isometry, and this is conjectured to be true in general. In this talk, we explore to what extent the marked length spectrum on a sufficiently large finite set approximately determines the metric.

Vaughn Climenhaga (University of Houston)
Title: *Hausdorff measure and the MME on general shift spaces.*
Abstract: For shift spaces, topological entropy is equal to Hausdorff dimension. I will describe how the corresponding Hausdorff measure gives the eigenmeasure and eigenfunction of the Ruelle-Perron-Frobenius operator (for one-sided shifts) and the conditional measures of the measure of maximal entropy (for two-sided shifts). I will also describe a similar construction for equilibrium states of nonzero potential functions, which is the symbolic version of an approach developed by myself, Pesin, and Zelerowicz for uniformly hyperbolic systems. The present results for shift spaces are joint work with Jason Day, and extend beyond shifts of finite type; the main hypotheses are uniform counting estimates and the presence of subsets with product structure. Finally, I will discuss applications such as piecewise expanding interval maps and Sinai billiards.

Emilio Corso (University of British Columbia)
Title: *Expanding homogeneous curves on compact surfaces: the exact equidistribution rate*
Abstract: A striking observation of Zagier, dating back to the early eighties, is that the Riemann hypothesis would follow from establishing the optimal rate at which expanding closed horocycles equidistribute on the modular surface. In the case of compact hyperbolic surfaces, where the underlying spectral theory is considerably less intricate, we obtain a precise asymptotic expansion for averages of sufficiently regular observables along geodesic translates of arbitrary homogeneous curves. The talk is mostly based on joint work with Davide Ravotti.

Dmitry Dolgopyat (University of Maryland, College Park)
Title: *Exponential Mixing implies Bernoulli.*
Abstract: Exponential mixing is perhaps the strongest chaotic property of dynamical systems. In this talk we present a proof of the theorem proven in a joint work with Adam Kanigowski and Federico Rodriguez Hertz and saying that a system enjoying exponential mixing with respect to a smooth measure is isomorphic to a Bernoulli shift.

Alena Erchenko (University of Chicago)
Title: *On thermodynamical formalism for CAT(0) spaces.*
Abstract: Consider a compact geodesically complete rank one locally CAT(0) space. There is a natural generalization of classical geodesic flow for such spaces. We will discuss progress on the existence and uniqueness of equilibrium states in this setting, their properties, and some open questions. This talk is based on joint work with Ben Call, Dave Constantine, Noelle Sawyer, and Grace Work.

Boris Hasselblatt (Tufts University)
Title: *Partial to hyperbolicity: the careers of Brin and Pesin.*
Abstract: An appreciation of their thorny road to mathematics and the ways in which they have advanced mathematics.

Michael Jakobson (University of Maryland, College Park)
Title: *SRB measure for some Henon-like diffeomorphisms.*
Abstract: A major part of the proof of existence of SRB measures for Henon-like families are inductive estimates of distortions and parameter dependence for iterates of the original map f. A goal is to construct a power map $F = \{F_i = f^{n_i}\}$ which has SRB measure ν. Then an f-invariant measure μ can be constructed from ν and ergodic properties of μ can be studied. We consider some piecewise smooth families of 2-d maps which are not small perturbations of 1-d maps, and prove existence of SRB measures for a set of parameters of positive Lebesgue measure.

Konstantin Khanin (University of Toronto)
Title: *Typical rotation number for families of circle maps with singularities.*
Abstract: I shall discuss how one can define in a natural way the notion of typical rotation numbers for families of circle maps with singularities. This problem is related to a well known fact that in the case of maps with singularities the set of parameters corresponding to irrational rotation numbers has zero Lebesgue measure. I shall also discuss a natural setting for the Kesten theorem in the case of maps with singularities.

Tamara Kucherenko (City College of New York)
Title: *Asymptotic behavior of the pressure function for Hölder potentials.*
Abstract: We study the behavior of the pressure function for Hölder potentials on mixing subshifts of finite type. The classical theory of thermodynamic formalism shows that such pressure function is convex, analytic and has slant asymptotes. We provide a sharp exponential lower bound on how fast the pressure function approaches its asymptotes. As a counterpart, we also show that there is no corresponding upper bound by exhibiting systems for which the convergence is arbitrarily slow. In addition, we determine that the pressure function of a Hölder potential satisfies a coarse uniform strict convexity property. Asymptotic bounds and quantitative convexity estimates are the first additional general properties of the pressure function obtained in the settings of Bowen and Ruelle since their groundbreaking work more than 40 years ago. (This is based on joint work with Anthony Quas.)

Francois Ledrappier (CNRS, Paris)
Title: *Dimension of Oseledets measure.*
Abstract: We consider a random walk on the group of dxd matrices. Oseledets Theorem yields a random family of subspaces. The Oseledets measure is the distribution of these spaces in the product of the corresponding Grassmannians. We assume that the random walk has countable support, first moment and finite entropy. Then, the Oseledets measure is exact-dimensional. We'll discuss some aspects of the proof. This is joint work with Pablo Lessa.

Stefano Luzzatto (Abdus Salam International Centre for Theoretical Physics.)
Title: *Statistical and non-statistical dynamics in doubly intermittent maps.*
Abstract: We introduce a large family of one-dimensional full branch maps which generalize the classical "intermittency maps" by admitting two neutral fixed points and possibly also critical points and/or singularities. We study the statistical properties of these maps for various parameter values, including the existence (and non-existence) of physical measures, and their properties such as decay of correlations and limit theorems. In particular we describe a new mechanism for relatively persistent non-statistical chaotic dynamics. This is joint work with Douglas Coates and Muhammad Mubarak.

Vaughn Osterman (University of Maryland, College Park)
Title: *Length Spectrum Rigidity in Dispersing Billiard Systems.*
Abstract: We consider the class of dispersing billiard systems in the plane formed by removing three convex analytic scatterers satisfying the non-eclipse condition. The collision map in this system is conjugated to a subshift, providing a natural labeling associating each periodic orbit with some coding σ. We study the problem of marked length spectrum rigidity for this class of systems, which asks whether or not a system is uniquely determined by the function that maps each coding σ to the perimeter of its respective periodic orbit. My result is that two such systems have the same marked length spectrum if and only if their collision maps are analytically conjugate to each other near a certain homoclinic orbit.

Jaime Paradela (Universitat Politecnica de Catalunya (UPC))
Title: *Unstable Dynamics in the Restricted 3 Body Problem.*
Abstract: We study the existence of unstable behavior in the Restricted 3 Body Problem (R3BP), which models the motion of a massless body under gravitational interaction with two massive bodies. In particular, we are interested in the existence of orbits which connect certain arbitrarily far regions of the phase space, in the spirit of what is usually referred to as Arnold Diffusion. The occurence of this kind of unstable behavior has been conjectured by Arnold himself to be "typical" in the complement of integrable systems. Despite presenting strong degeneracies, we construct diffusive orbits in the R3BP: more concretely, we build orbits along which the angular momentum of the massless body (a conserved quantity for the 2 Body Problem) experiments arbitrarily large variations. This is joint work with Marcel Guardia (UB) and Tere M. Seara (UPC).

Françoise Pène (University of Brest)
Title: *Probabilistic limit theorems for the periodic Lorentz gas.*
Abstract: The periodic Lorentz gas is a natural example of dynamical system preserving an infinite measure. Because of its periodicity, this system can be represented by a Z^d-extension over the Sinai billiard. Thus, the ergodic properties of the periodic Lorentz gas are closely related to the chaotic behaviour of the Sinai billiard, and in particular with the local limit theorem established by Domokos Szász and Tamás Varjú. When the horizon is finite, the free flight is bounded, and powerful tools can be used to establish strong results, such as quantitative recurrence results, expansions in mixing, limit theorems for Birkhoff sums, for non-stationary Birkhoff sums and for solutions of perturbed differential equations (results in collaboration with Benoît Saussol, with Dima Dolgopyat and Péter Nándori, with Damien Thomine, results by Maxence Phalempin). Finally we will also state results in the more difficult case of the infinite horizon (results in collaboration with Dalia Terhesiu, and also with Ian Melbourne).

Eduardo Oregon Reyes (University of California, Berkeley)
Title: *Continuity of Bowen-Margulis currents for hyperbolic groups.*
Abstract: For a closed, negatively curved manifold, the geodesic flow has a unique probability measure of maximal entropy: the Bowen-Margulis measure. By a theorem of Katok-Knieper-Pollicott-Weiss, the Bowen-Margulis measure is continuous under perturbations of the Riemannian metric. Instead of the fundamental group of this manifold, we can consider an arbitrary word-hyperbolic group, so that the manifold and its metric are replaced by a proper and cobounded isometric action on a geodesic metric space. For each of these isometric actions, Furman used Patterson-Sullivan measures to construct a geodesic current analogous to the Bowen-Margulis measure, now an invariant Radon measure on the space of pairs of distinct points in the boundary at infinity. In this talk, I will explain a version of Katok-Knieper-Pollicott-Weiss's theorem in the context of word-hyperbolic groups: continuous perturbations of the isometric action give a continuous perturbation of the Bowen-Margulis current.

Yuping Ruan (University of Michigan, Ann Arbor)
Title: *Filling volume minimality and boundary rigidity of metrics close to a rank-1 symmetric metric.*
Abstract: A compact manifold with a smooth boundary is boundary rigid if its boundary and boundary distance function uniquely determine its interior up to boundary preserving isometries. Under certain natural conditions, the notion of boundary rigidity is closely related to Gromov's filling minimality. In this talk, we will first give a brief overview of Burago-Ivanov's approach to prove filling minimality and boundary rigidity for almost Euclidean and almost real hyperbolic metrics. Then we will explain how we generalize their results to regions in a rank-1 symmetric space equipped with an almost symmetric metric. If time permits, we will also explain the relations to Besson-Courtois-Gallot's barycenter constructions used in their celebrated volume entropy rigidity theorem.

Pablo Shmerkin (University of British Columbia)
Title: *Beyond Furstenberg's intersection conjecture.*
Abstract: Hillel Furstenberg conjectured in the 1960s that the intersections of closed $\times 2$ and $\times 3$-invariant Cantor sets have "small" Hausdorff dimension. This conjecture was proved independently by Meng Wu and by myself; recently, Tim Austin found a simple proof. I will present some generalizations of the intersection conjecture and other related results.

Michael Shub (City College of New York)
Title: *Uses of the double fibration technology in dynamics.*
Abstract: I will survey two topics:
1. The relation between average Lyapunov exponents and random exponents. Recently Diego Armentano, Gautam Chinta and Siddhartha Sahi have extended what is known about $GL(n, \mathbb{C})$ to $GL(n, \mathbb{R})$. This revives interest in the general case.
2. The average number of points of intersection between submanifolds of complimentary dimension of a closed manifold. This generalizes work I did with Smale in the 1990's and may be useful in dynamics. This is joint with with Axel Kodat and others.

Domokos Szász (Budapest University of Technology and Economics)
Title: *Sinai-type billiards and statistical physics.*
Abstract: Statistical physics motivated several beautiful problems for mathematics while also inspiring highly efficient theories and methods. Sinai-type billiards serve an excellent example. It is sufficient to mention projects aiming at (1) the Boltzmann-Sinai ergodic hypothesis (1872 vs. 1970), (2) Einstein's theory of Brownian motion (1905) and (3) understanding energy transfer with the aim to create a rigorous bedrock for Fourier's 1822 law of heat conduction. In the talk I will mainly focus on item (2) though tangentially also discuss topics (1) and (3).

Zhenqi Wang (Michigan State University)
Title: *Local rigidity of higher rank partially hyperbolic algebraic actions.*
Abstract: We give a complete solution to the local classification program of higher rank partially hyperbolic algebraic actions. We show local rigidity of abelain ergodic algebraic actions for symmetric space examples, twisted symmetric space examples and automorphisms on nilmanifolds. The method is a combination of representation theory, harmonic analysis and a KAM iteration. A striking feature of the method is no specific information from representation theory is needed. It is the first time local rigidity for non-accessible partially hyperbolic actions has been obtained other than torus examples. Even for Anosov actions, our results are new: it is the first time twisted spaces with non-abelian nilradical have been treated in the literature.

William Wood (University of California, Irvine)
Title: *Uniform Hyperbolicity and the Periodic Anderson-Bernoulli Model.*
Abstract: In this talk we will focus on the notion of uniform hyperbolicity of sets of matrices, and apply it to transfer matrices related to a discrete Schrodinger operator to study its spectrum. Specifically, we will show how to apply Johnson's Theorem, which claims that a Schrodinger cocycle is uniformly hyperbolic if and only if the corresponding energy value is not in the almost sure spectrum, to the periodic Anderson-Bernoulli Model. As a result, we will prove that the spectrum of period two Anderson-Bernoulli Model consists of at most four intervals. A period 3 model, given specific conditions, can have infinitely many intervals in the spectrum, however.