MICHAEL BRIN AND YAKOV PESIN

Lecture given by Boris Hasselblatt on November 3, 2022

It is a pleasure to start this session with a few words about the careers of the two friends whose birthdays this gathering celebrates. Michael Brin and Yakov Pesin, or Misha and Yasha for their friends, have been an influence on me in distinctive ways, and both have a large footprint in dynamical systems.

Misha was the first to teach me a course on dynamical systems; this was at the University of Maryland in the 1983-84 academic year. And with Yasha, I have coauthored a research paper, a book chapter, and a brief biography. That I first met each of them at such different stages of my own career owes to an instance of sensitive dependence on inital conditions to which I will come later.

The more perceptive members of this audience may have guessed by now that Yasha and Misha are close in age. They were born at a time which for some period looked rather auspicious for a career in mathematics. Both attended elite mathematical high schools and graduated with honors in 1965. In fact, Yasha attended a high school run by Kolmogorov and even was the TA for Kolmogorov's class in that high school. Misha had from childhood dreamed of becoming an astrophysicist, but in the USSR this specialty was not available to Jews because as he put it, physics was a military science and the Jews were not considered trustworthy enough.

Both also passed the entrance exams to Mekh–Mat, the Moscow State University Faculty of Mechanics and Mathematics, where gaining acceptance was exceedingly difficult. Discrimination there was administered by means of entrance exams for which Jews were tested in different rooms from other applicants and graded more harshly.

They entered right in the middle of the decade which is generally considered the golden age of Moscow mathematics. Accordingly, their core

---

This text is not a work of original scholarship. It incorporates unacknowledged quotes or paraphrasing from published sources authored by Michael Brin, Anatole Katok, Svetlana Katok, Mark Malseed, Yakov Pesin.

mathematics courses were taught by distinguished mathematicians such as Efimov, Manin, Arnold, Vishik, Shabat, and Shilov, and during their 3rd through 5th years they greatly benefited from many topics courses and special seminars offered by stellar faculty. One should note that "undergraduate" education in Moscow went for 5 years of which the last 2 were what in the US would be considered graduate level, leaving students ready to pass US qualifiers in virtually any mathematical subject area.

Their undergraduate adviser was Yakov Sinai, and they naturally started to attend the Sinai–Alexeyev seminar which was then the central arena for those interested in dynamics. But the only course work in dynamical systems either of them ever had were topics courses in dynamical systems and ergodic theory taught by Anatole Katok, known as Tolya to his friends, who got his doctorate with Sinai in 1968 and on his own initiative gave these classes. In the Fall of 1969 both started attending the Anosov–Katok seminar, which became their sole connection to mathematics for a decade or two.

Maybe a few words about golden ages will help put subsequent events in context. As Anatole and Svetlana Katok wrote, the 1920s were the first golden age of mathematics in the Soviet Union. The comparatively favorable conditions during the 1920s started to change in the 1930s after Stalin consolidated his power, but the continued flourishing of Soviet mathematics during the 1930s and 1940s presents a dramatic contrast to the tragic state of the society in general. In 2009,the journalist Masha Gessen explained the fact that Soviet mathematics had not been decimated as follows. *"Three factors saved math. First, Russian math happened to be uncommonly strong right when it might have suffered the most, in the 1930s. Second, math proved too obscure for the sort of meddling Joseph Stalin most liked to exercise: It was simply too difficult to ignite a passionate debate about something as inaccessible as the objective nature of natural numbers (although just such a campaign was attempted). And third, at a critical moment math proved immensely useful to the state."* After the death of Stalin in 1953, the most grotesque elements of his policies were quickly alleviated. The 1960s was the second golden age of mathematics in the Soviet Union.

Mathematics in the Soviet Union became even less welcoming to Jews in the late 1960s. The mathematics department at MSU saw increased

antisemitism and general oppression against liberal thought, and almost no Jews were accepted as either undergraduate or graduate students or faculty. For instance, upon his graduation, Katok was recognized as a prominent mathematician and definitely fit for a position either at the Moscow State University or at the Steklov Institute, but instead he had to consider himself most fortunate to land at the Central Economics–Mathematics Institute of the USSR Academy of Science (CEMI), where he was rather free to do research of his choice. Just a few years later, even the option for Misha and Yasha to go to graduate school closed. Although they graduated from Mekh-Mat with honors and were recommended for graduate school by Sinai and by the Mekh-Mat administration, the department communist party bureau rejected their applications. As Misha explained in an interview, *"I had all A's except for three classes where I got B's: history of the Communist Party, military training, and statistics, But nobody would even consider me for graduate school because I was Jewish. That was normal."* So, neither Yasha nor Misha were even recommended to the graduate program called "aspirantura" because of the new political headwinds.

In the end, Yasha landed at the Research Institute of Optical-Physical Measurements, while Misha got a junior position at the Research Economics Institute of the State Planning Committee GOSPLAN, considered a good job. In his words: *"I was trying to prove that, in a few years, living standards in Russia would be higher than in the United States, and I proved it. I know enough about math to prove whatever you want."* He could have made a career there, including a Ph.D in economics. Unlike at CEMI, this was a real economics job, though, and Misha did not greatly enjoy it. So he asked Katok a couple of years in, whether if he went into mathematics, there would a place and a job for him. Katok said yes—something one might consider almost foolhardy in the circumstances. As he wrote later: *"Sometimes my actions verged on irresponsibility... In retrospect it is clear that my fortunate circumstances engendered a not fully justified optimism which influenced Brin and Pesin whose circumstances were quite a bit less favorable. A more objective hard-headed view would probably have discouraged them and terminated their mathematical careers... While Brin and Pesin produced their work under difficult and unfortunate conditions, they at least were fortunate with its publication... having their outstanding work appear in a premier journal."*

Misha and Yasha faced a hard choice—to quit mathematics like many others or to combine it with their numbing full-time jobs. The seminars were at the heart of Moscow mathematical culture and a vital connection to the research enterprise for Misha and Yasha and others like them who were otherwise utter outsiders with no standing at these institutions. As Katok wrote: *"In the case of Pesin this was aggravated by geographical difficulties: He lived pretty far on the outskirts of Moscow and his job was located in another faraway corner of the city. Just coming to the seminar involved many hours of traveling by public transportation and he usually looked very tired."* The Brins were better off than many Muscovites who still lived in communal apartments. They occupied a tiny, three-room apartment in central Moscow, 350 square feet shared with Misha's mother.

The role of the Alekseev–Sinai and Anosov–Katok seminars was twofold: In addition to talks about research by participants and guests, many talks were about works by other mathematicians, mostly foreigners, very often based on preprints, which were rare and precious. They served as incubators of a staggering amount of mathematics. They typically met weekly in the afternoon or evening, with well-defined start times and much less well-defined end times, though officially they were scheduled for some two hours duration. For two decades, they were the main engine that put Moscow at the forefront of developing the modern theory of dynamical systems.

Katok could not serve as the adviser of record for Yasha and Misha because this would have combined a Jewish student with an adviser who was also Jewish and at an economics institute rather than a university. Instead, Anosov served as such. According to the accepted mode of operation in the Moscow community, he did not propose a problem or provide much guidance, but he was an important listener, and his comments were important—as were his later efforts to help Yasha defend his dissertation.

Katok proposed to Misha and Yasha that they work out what happens if in addition to expanding and contracting subbundles there is a subbundle with less contraction or expansion, and he pointed out that the frame flow on a manifold of negative curvature is a natural example of this situation. He gave them a pertinent paper by Sacksteder. Over the course of two years, they proceeded to develop the theory of partially hyperbolic dynamical systems. This is a well-recognized accomplishment,

and it is incredible that they managed this while only being able to work after hours or on weekends, taking time away from their families, which required understanding, sacrifice and strong support of their wives, who themselves held full-time jobs. Natasha Pesin was a senior editor in the division of mathematics at the "Education" Publishing House and Eugenia Brin worked in a research lab of the Soviet Oil and Gas Institute.

Katok and Anosov listened and commented, but did not get otherwise involved. When asked whether this work would be enough for dissertations, Anosov told them that they needed to say who proved what, that what they had would only be sufficient for non-Jewish candidates; Jews would need an additional paper. Misha proceeded to study frame flows as an application of partial hyperbolicity , and as Yasha was casting about for a project, he went back to a book on differential equations "with nonzero exponents," which Anosov had given him along with the suggestion that it might be useful. Together with the motivation to extend ergodicity of geodesic flows of compact surfaces from metrics with negative curvature to those without focal points, this was the origin of his theory of nonuniform hyperbolicity.

His first approach assumed that while the contraction and expansion rates may vary discontinuously with the orbit, the attached constant would be uniform. He presented this to Anosov as an add-on to the Brin–Pesin theory, and Anosov saw right away that this could not be right, sending Yasha back to the drawing board. On the two-hour subway ride home, Yasha figured it out, wrote up a draft at home and called Anosov with the good news. Anosov did not buy it and said that he was not willing to listen to more nonsense, but as a last chance he gave Yasha five minutes to convince him. Eventually, Yasha presented this work in the seminar, and it became clear that this was both correct and important. Yasha then wrote the Uspehi paper (his third paper and second major one), and Katok discussed drafts generously and helped with the exposition. Katok's promotion of this work continued past its completion. As he wrote: *"Of course, Pesin was not able to lecture about his work outside of the Soviet Union so the task fell on me when I found myself in 1978 first for six months in Western Europe and then in the United States."*

Following the traditional European model, one could obtain a doctorate by writing a dissertation and then finding a university where one would

be allowed to defend it. The latter part was the difficulty. In Misha's case, even Kolmogorov intervened with the head of Mech-Mat at the Moscow State University to let Brin defend there, but to no avail. Misha managed to instead use a private connection to Naum Akhiezer in Kharkiv, where he defended with Anosov as adviser of record and Arnold and Lyubich as opponents. He recalls Arnold complaining about his handwriting of the formulas in the text.

Life didn't change much even after he received his Ph.D. He continued in his day job at GOSPLAN and received a 100-ruble raise. "I thought I was rich. Life was beautiful," he said in a later interview.

Arranging Yasha's defense took another four years of exertions, until 1979, when his work was already world-famous, and it was entirely due to Anosov that he was able to defend his dissertation in Gorky (now Nizhnij Novgorod) with support from Leonid Shilnikov. Here is the story as told by Yasha and Misha themselves: *"After several unsuccessful attempts to find a place for the defense, Anosov made a deal with O, the head of the Scientific Council at Gorky. O had two graduate students with theses ready to be defended and he agreed to arrange the three defenses on the same day in exchange for Anosov ensuring a safe passage of the theses through the Higher Attestation Board. For Anosov it was a rare and very serious compromise with his principles, but he decided it was worth it. As he put it "Yasha, I traded you for two." As it turned out the thesis of one of O's students claimed three theorems of which two were completely wrong and the third one needed very serious corrections. As a result that student dropped out and, as Anosov remarked: "it became a fair trade." The deal notwithstanding, the positive outcome was not guaranteed. Unexpected help came from Evgeniya Aleksandrovna Leontovich-Andronova, the widow of the famous mathematician Andronov and a prominent member of the Scientific Council. At the end of the defense proceedings she said: "When my husband was alive, this work would result in a Doctor of Science degree, years later it would be considered an outstanding PhD thesis, and now we are thinking whether to vote yes or no." Actually, there were still two negative votes."*

In 1972 the Banach Center in Warsaw was established with the aim to promote and stimulate international cooperation in mathematics, especially between the East and West. Brin and Katok visited in 1977 for an

international conference that provided a major encounter between "the East" and "the West." This was the height of the period when Jewish scientists and mathematicians could leave the Soviet Union, and the experience got Misha thinking. For the first time, he had been able to mingle freely with colleagues from the United States, France, England and Germany. Discovering that his intellectual brethren in the West "were not monsters," he listened as they described the opportunities and comforts of life beyond the Iron Curtain. He came home and announced that it was time for the family to emigrate. They knew the perils of applying for an exit visa. They could easily end up refuseniks, unable to find work, shunned, in perpetual limbo. Nobody had promised Misha a position abroad but he was confident he could find work in the West that was intellectually stimulating and would support the family. Genia, however, was unconvinced. They had lived in Moscow their entire lives. They had decent jobs and a young son. Was it worth it to try to leave? "I didn't want to go," she said in an interview. "It took a while for me and his mother to agree. I had a lot more attachments." It was up to Misha to do the convincing. "I was the only one in the family who decided it was really important to leave—not in some distant future," he said.

For Genia Brin, life in Moscow was comfortable. She, too, had managed to overcome the entrance hurdles to attend Moscow State, graduating from Mech-Mat. In a research lab of the Soviet Oil and Gas Institute, a prestigious industrial school, she worked alongside a number of other Jews. She was content in her job and had many friends. The Brins' encounters with institutional anti-Semitism did not extend to day-to-day interactions with colleagues and neighbors. Highly assimilated into Russian culture, they were part of the intelligentsia and had a circle of university-educated friends.

For Genia, the decision ultimately came down to their son Sergey. While Misha was thinking as much about his own future as his son's, for her, "it was 80/20 about Sergey." In 1978, Katok arranged an official invitation for the Brins from fictitious relatives in Israel, which they received that summer, and which was required for permission to emigrate. They used this invitation to apply for permission to emigrate to Israel in September 1978, and Misha was promptly fired. Genia, who had obtained her job through a relative, had to quit to insulate him from any recrimination.

As she tells it: "When he got a whiff of our intentions, he said 'please get out of there as soon as possible.' It had to be a secret from everybody at work, my real reason for leaving. So I lied to all of my coworkers that I was simply leaving my job because I got another job, where I would only have to be at work three days a week and the salary would be higher. I made up—totally made up—the name of a place where I was planning to work." There was no other job, of course, and suddenly they found themselves with no income. To get by, Misha translated technical books into English, but it was painstaking work. He also began to teach himself computer programming, having no expectation of getting an academic position if they ever got out. When Genia found temporary work, again lying about her situation, they shared responsibility for looking after Sergey, who stayed at home rather than attend a miserable Soviet pre-school. And then they waited. For many Soviet Jews, exit visas never came. But, in May 1979, the Brins were granted papers to leave the U.S.S.R. "We hoped it would happen," Genia said, "but we were completely surprised by how quickly it did." The timing was fortuitous: They were among the last Jews allowed to leave until the Gorbachev era.

Sergey, who turned six that summer, remembers what followed as simply "unsettling"—literally so. "We were in different places from day to day." The journey was a blur. First Vienna, where the family was met by representatives of the Hebrew Immigrant Aid Society, which helped thousands of Eastern European Jews establish new lives in the free world. Then, on to IHES near Paris, where Anatole Katok had arranged a temporary research position for Misha. Katok furthermore utilized the upcoming special year in Maryland to get Misha hired there on a visiting (later permanent) position. Thanks to his mother, an English teacher, Misha already knew the language perfectly. And Genia found employment as a researcher at the Goddard Space Flight Center.

When the family finally landed in America on October 25, they were met at New York's Kennedy Airport by friends from Moscow. Sergey's first memory of the United States was of sitting in the backseat of the car, amazed at all the giant automobiles on the highway as their hosts drove them home to Long Island. The Brins found a house to rent in Maryland—a simple, cinder-block structure in a lower-middle-class neighborhood not far from the university campus. With a $2,000 loan from the Jewish

community, they bought a 1973 Ford Maverick. And, at Katok's suggestion, they enrolled Sergey in Paint Branch Montessori School in Adelphi.

Sam, the younger son, graduated from the University of Maryland in 2009 with a BS in Computer Science and minor in Physics. He began his career as a software engineer for a research group at Massachusetts General Hospital before transitioning to the startup space.

Like many others, the Pesins were less fortunate and did not receive permission to emigrate before the doors slammed shut for a decade as the war in Afghanistan started in 1980—and Yasha was stuck in a job even less desirable than Misha's. Joel Lebowitz invited him to visit IHES in 1988. Knowing full well that the Soviet authorities would typically deny travel permits for conferences "because there is not enough lead time," he had made it an open invitation, and thanks to this and *glasnost*, Yasha was actually allowed to go. In what sounds like a trope from crime fiction, the IHES allowed guests one free call outside Paris, and Yasha used this to call Misha in Maryland—they had not talked for a decade. Generous to a fault, the first thing Misha said was "What's your number, I'll call you back!" Yasha spoke of wanting to emigrate, and Misha said he should send a CV. Yasha sent whatever he thought that meant, and between them, Misha and Anatole Katok made it usable and sent it to Bob Zimmer, then chair of mathematics at the University of Chicago. When the Pesins had received permission to emigrate and started preparations, the University of Chicago called to say that Yasha had a (temporary) job there starting January 1, 1990, and asked when they were going to arrive. They had no idea but in the end made it there by January 23, finding a month's salary waiting for them, and support from the Zimmers well beyond the call of duty. Along the way Yasha had been invited to talk in Karl Sigmund's seminar in Vienna, where he also met Josef Hofbauer and Mitchell Feigenbaum; Sigmund's financial support made them "wealthy" enough to even support some indigent emigrants in turn. Then Laura Tedeschini-Lalli called from Rome (at the instigation of Misha) with an offer of a two-month appointment—and it turned out that they spent three.

From Chicago, the Pesins visited the Katoks (and Caltech) in February—at the very time Katok was negotiating with the Pennsylvania State University, and by early March there was an offer for Yasha of a permanent position at Penn State. Since several other institutions were thinking about

making him an offer, he consulted with Misha, who told him to go to Penn State: "Tolya will be there, and he'll organize everything, and you can just keep doing mathematics."

My plan for the next three hours is to describe some of what Misha and Yasha have done to advance mathematics.

I will try to organize Misha's works into a few strands in a vaguely chronological fashion.

Misha's first paper, in 1972, showed that Morse–Smale diffeomorphisms are generically not time-1 maps; this and more was independently shown by Palis. In 1996 Misha returned to the subject with Yasha to prove among other things that being Morse–Smale is a C^1-open condition.

A 1974 paper on "A lower bound for the entropy of a smooth dynamical system" gave an inequality of Ruelle–Margulis type.

In 1974 Yasha and Misha published the original paper "Partially hyperbolic dynamical systems" in Izvestia, and in the following year, Misha published "Topological transitivity of a certain class of dynamical systems, and flows of frames on manifolds of negative curvature" in Functional Analysis and its Applications and "The topology of group extensions of C-systems" in Zametki. For the less grizzled audience members I will explain that " C-systems" was the original term for "Anosov systems."

Misha digressed significantly to investigate which manifolds admit Anosov diffeomorphisms: "Nonwandering points of Anosov diffeomorphisms" and "A fundamental group of a manifold that admits an Anosov diffeomorphism" appeared in Doklady. After his emigration, "On the spectrum of Anosov diffeomorphisms" was published in the Israel Journal of Mathematics in 1980, and "Anosov diffeomorphisms with pinched spectrum" with Manning in a Warwick proceedings volume in 1981.

The invitation to the IHES arranged by Katok helped the visibility of Brin's work and resulted in a collaboration with Gromov, who often visited the IHES and permanently joined it in 1982. This began a new sequence of papers on frame flows; "On the ergodicity of frame flows" with Gromov appeared in Inventiones in 1980. The survey "Ergodic theory of frame flows" in the proceedings of the special year which had made the invitation to Maryland possible, and "Frame flows on manifolds with pinched negative curvature" with Karcher appeared in 1984. Others have been honing those results even quite recently.

At the same time, Misha joined efforts to produce Bernoulli diffeomorphisms: "Bernoulli diffeomorphisms and group extensions of dynamical systems with nonzero characteristic exponents" with Feldman and Katok appeared in the Annals in 1981 and "Bernoulli diffeomorphisms with $n-1$ nonzero exponents" in the inaugural issue of Ergodic Theory and Dynamical Systems in 1981. I will note a 2004 counterpart "Every compact manifold carries a hyperbolic Bernoulli flow" by Yasha together with Huyi Hu and Anna Talitskaya.

Also in the same year, the famous definition of local entropy with Katok almost surreptitiously appeared in IMPA conference proceedings.

The first papers of Misha's which I read, and probably the first research papers by anyone that I read in any depth, concern the structure of geodesic flows, and they came about in collaboration with Ballmann and others: "On the ergodicity of geodesic flows" appeared in Ergodic Theory and Dynamical Systems in 1982 and showed that the geodesic flow of a rank-1 Riemannian manifold of nonpositive sectional curvature is ergodic and Bernoulli. The monumental twin Annals papers "Structure of manifolds of nonpositive curvature. I&II" with Ballmann and variously Eberlein and Spatzier were the foundation for the subsequent classification of higher-rank such manifolds by Ballmann and independently by Burns and Spatzier.

A little later, Misha investigated related issues in 1987 twin papers in the Journal of Differential Geometry with Ballmann and Burns: "On surfaces with no conjugate points" showed that Busemann functions may be discontinuous, and "On the differentiability of horocycles and horocycle foliations" established that C^2 regularity of these is sharp.

And, in the 1990s he embarked on subjects one now places with geometric group theory, together with Ballmann: "Polygonal complexes and combinatorial group theory" in 1994, "Orbihedra of nonpositive curvature" in 1995, "Diameter rigidity of spherical polyhedra" in 1999, and "Rank rigidity of Euclidean polyhedra" in 2000.

From the late 80s onwards, collaborations with Yuri Kifer produced several "stochastic" papers: "Dynamics of Markov chains and stable manifolds for random diffeomorphisms" in 1987, "Harmonic measures on covers of compact surfaces of nonpositive curvature" in 1993, "Brownian motion and harmonic functions on polygonal complexes" in 1995 (note

the connection with geometric group theory), "Brownian motion, harmonic functions and hyperbolicity for Euclidean complexes" in 2001, and also a paper with Freidlin "On stochastic behavior of perturbed Hamiltonian systems" in 2000, which studies one-dimensional nonlinear Hamiltonian oscillators subject to weak deterministic perturbations.

20 years ago, as the subject began a revival, Misha returned to partial hyperbolicity with "On dynamical coherence" in 2003 and two papers with Burago and Ivanov: "On partially hyperbolic diffeomorphisms of 3-manifolds with commutative fundamental group" in a 2004 volume for Katok's 60th birthday, and "Dynamical coherence of partially hyperbolic diffeomorphisms of the 3-torus" in 2009.

As you see, his collaborations spanned the globe, so it is well to point specifically to Maryland collaborators. Misha worked with William Ott and Jim Yorke on the lovely paper "Enveloping manifolds," which appeared in 2004. But most prominently, there is the famous 2002 book "Introduction to dynamical systems" which he wrote with Garrett Stuck. Misha personally promised me that the 2015 edition is wholly devoid of any errors!

I would like to give a like account of the many research contributions Yasha has made, but the wide range of his over 100 publications defeats me. Accordingly, I will discuss the works from his Moscow days and provide a very few limited later highlights.

I previously mentioned Yasha's works undertaken jointly with Misha, and I highly recommend reading Yasha's 2004 book "Lectures on partial hyperbolicity and stable ergodicity" in the Zürich Lectures in Advanced Mathematics which makes the Brin–Pesin theory *accessible*, as it were, as well as the Pugh–Shub approach, which brought about the resurgence of the subject.

But first, his 1973 paper "The existence of invariant foliations for a diffeomorphism of a smooth manifold" studies the Mather spectrum, its stability, and implications for partial hyperbolicity and invariant foliations. I should point to the remarkable related 1995 paper "On the integrability of intermediate distributions for Anosov diffeomorphisms" with Jiang and de la Llave.

In the next year, "The behavior of the solutions of a certain strongly nonlinear differential equation with retarded argument" studied periodic and stationary solutions of delay differential equations and their stability.

He returned to the subject in 1980 with "Stability and periodic solutions of some systems of differential equations with lag."

In the spirit of the Franks–Williams construction, Yasha gave "An example of a nonergodic flow with nonzero characteristic exponents," also in 1974.

And then, the papers on nonuniform hyperbolicity and geodesic flows started appearing: "Characteristic Ljapunov exponents, and ergodic properties of smooth dynamical systems with invariant measure" and "Families of invariant manifolds that correspond to nonzero characteristic exponents" in 1976. Then, in 1977, "Characteristic Ljapunov exponents, and smooth ergodic theory" in Uspehi and "Geodesic flows in closed Riemannian manifolds without focal points" in Izvestia. In the same year "A description of the π-partition of a diffeomorphism with an invariant measure" as the finest partition subordinate to the stable foliation. Then, famously, "Formulas for the entropy of the geodesic flow on a compact Riemannian manifold without conjugate points" in 1978 and the magisterial 1980 Uspehi survey "Geodesic flows with hyperbolic behavior of trajectories and objects connected with them" plus a broader 1981 survey with Sinai of hyperbolic dynamics, entitled "Hyperbolicity and stochasticity of dynamical systems." In 1985, Yasha also wrote a chapter on "General theory of smooth hyperbolic systems" in the Springer Encyclopedia of Mathematical Sciences.

Also with Sinai, he studied "Gibbs measures for partially hyperbolic attractors," showing that any absolutely continuous measure asymptotically pushes forward to a Gibbs measure on the attractor which is absolutely continuous on unstable manifolds.

You can discover much of this with a simple MathSciNet query. Now is a good time to digress to a major contribution that is less apparent.

In 1977, Giancarlo Benettin and Jean-Marie Strelcyn had submitted a study of the "stochastic transition" in a one-parameter family of plane billiards called "generalized stadium" or Benettin–Strelcyn billiard that interpolates between the Bunimovich stadium and the circle as follows: circle arcs surmount the sides of a square, matching tangent lines at the corners. Strelcyn asked Katok whether the entropy formula could be applied in this context. As Katok and Yasha thought how to extend Pesin theory to systems with discontinuities, such as these billiards, their trips to the

Anosov seminar provided opportunity for discussion. Part of the itinerary was a 30-minute bus ride, and one day Yasha realized on this bus that the missing sufficient condition (beyond previously known assumptions on derivatives) was that the *volume* of a neighborhood of the discontinuity set be bounded by a power of the *thickness* of that neighborhood. Just then, the passengers were asked to produce their tickets, and the pair realized that in their mathematical excitement they had completely forgotten to buy any. The fine was some 10 times as much as the bus fare, and Katok declared that "this is the price of a new mathematical discovery."

Because the impending departure of the Katoks in 1978 made it infeasible to collaborate on writing this result up together, they agreed that Katok would publish it himself abroad. Katok wrote to Strelcyn that undoubtely the entropy formula can be applied to billiards but that the proof for this case had to be written. The resulting Katok–Strelcyn book has since been the standard reference on hyperbolic systems with singularities, and Yasha soon generalized this to dissipative situations.

I will mention in passing a famous 1984 paper Yasha wrote with Pitskel on "Topological pressure and the variational principle for noncompact sets," which extended the Bowen–Walters theory beyond compactness.

In 1982, Yasha started building the dimension theory in dynamical systems with "A formula for the Hausdorff dimension of a two-dimensional hyperbolic attractor" and three papers in 1984 alone: "Estimation of the Hausdorff dimension of a basis set in a neighborhood of a homoclinic trajectory" with Afraimovich, "On the notion of the dimension with respect to a dynamical system," and "Ergodic properties and the dimension of Lorenz attractors." The next year saw his seminal paper "A generalization of Carathéodory's construction for dimensional characteristic of dynamic systems" in a Birkhäuser volume, and the Proceedings of the 1984 ICM announced related results as well as the generalization of the Katok–Strelcyn results to disspative situations; the full paper appeared in Ergodic Theory and Dynamical Systems in 1993. A second major paper with Afraimovich in Mathematical Physics Reviews in 1987 treats "Dimension of Lorenz type attractors," followed by Yasha's large 1988 Uspehi paper on "Dimension-like characteristics for invariant sets of dynamical systems." and a 1990 paper on "Dimension-like characteristics of generalized-hyperbolic attractors."

Maybe it is good to remind ourselves that all this happened while Yasha held a full-time job unrelated to mathematics!

From 1990 onward, Yasha was actually paid to do mathematics, and his works become too numerous to recite. We will have to settle for a few highlights. These include much more work on dimension theory, beginning with numerous works on multifractal analysis variously with Barreira, Sadovskaya, Schmeling, and Howie Weiss. With Luis Barreira and Jörg Schmeling, he wrote the 1999 Annals paper "Dimension and product structure of hyperbolic measures," which proves the Eckmann-Ruelle conjecture that the Hausdorff dimension of a hyperbolic measure is the sum of the Hausdorff dimension of its stable and unstable slices. This research program continues to the present, such as with the 2019 GAFA paper "Dimension estimates for non-conformal repellers and continuity of subadditive topological pressure" with Yongluo Cao and Yun Zhao. Most prominently among the outcomes of this program is the now standard reference, the wonderful 1997 book "Dimension theory in dynamical systems" in the Chicago Lectures in Mathematics series. This one is associated with fond memories I have of a semester Yasha and I and our wives shared in Zürich.

Yasha and I have also coauthored three mutually different works. A brief 2004 biography of Anatole Katok for a birthday volume, a 2006 Handbook chapter on "Partially hyperbolic dynamical systems," and a 2014 paper with Jörg Schmeling to the effect that "Pointwise hyperbolicity implies uniform hyperbolicity."

Twenty years ago Yasha established the pervasiveness of hyperbolicity in the sense that "Every compact manifold carries a completely hyperbolic diffeomorphism," a paper with Dmitry Dolgopyat in 2002, and "Every compact manifold carries a hyperbolic Bernoulli flow," a book chapter with Hu and Talitskaya in 2004.

With coauthors, Yasha also established "The essential coexistence phenomenon" in several contexts, i.e., the coexistence of sets of positive measure of hyperbolicity and nonhyperbolicity.

A rather large subject in importance and number of publications encompasses thermodynamic formalism, equilibrium measures, phase transitions, and Sinai-Ruelle–Bowen measures.

Two papers treat scaled entropy and q-entropy, and numerous others study dynamical systems arising from applications.

More recent works on issues related to partial hyperbolicity are "Non-absolutely continuous foliations" with Hirayama in 2007, and "Stable ergodicity for partially hyperbolic attractors with negative central exponents" with Burns, Dolgopyat, and Pollicott in 2008.

Yasha is a prolific book author, and the Chicago and Zürich Lectures volumes are but two of six books he has published. A related one is "Lectures on Fractal Geometry and Dynamical Systems," which he wrote with Vaughn Climenhaga for the AMS Student Mathematical Library.

As a light counterpart and being here at Penn State, one should mention the chapter "Fractals and dynamics" in the 2003 MASS Selecta volume. And three additional books on nonuniform hyperbolicity plus major surveys were coauthored with Luis Barreira. A prelude was the 2001 book chapter "Lectures on Lyapunov exponents and smooth ergodic theory" in what is called the "Seattle Proceedings." In 2002 the book "Lyapunov exponents and smooth ergodic theory" appeared in the AMS University Lecture Series, followed by the 2006 book-length Handbook chapter "Smooth ergodic theory and nonuniformly hyperbolic dynamics." The high point was the celebrated and definitive 2007 text and reference "Nonuniform hyperbolicity" with Cambridge University Press, which puts it all together. Later, the two decided to follow it with a lighter 2013 text "Introduction to smooth ergodic theory" in the AMS Graduate Studies in Mathematics series.

Mathematics can be advanced in many ways beyond proving theorems and publishing, and both Misha and Yasha have done much more, such as mentoring doctoral students, organizing conferences, serving as editors and reviewers. And Yasha now directs the Anatole Katok Center for Dynamical Systems and Geometry.

This is a very good time to digress to a story I heard from Alicia Nitecki some time back. She and my colleague Ziggy Nitecki lived in the DC area in the 1982–83 academic year when Ziggy served at the National Science Foundation as Program director for geometric analysis. On one of their visits to the Brin household, Alicia happened upon little Sergey Brin in front of a computer. Ever disdainful of technology, she asked him "just what is so great about computers?" He looked up at her and said without hesitation "you can put all the information in the world on them!"

He just about has done so, and this put Misha in a position to support mathematical endeavors financially.

You know or will presently learn about The Michael Brin Prize in Dynamical Systems and the Michael Brin Dynamical Systems Prize for Young Mathematicians. Four years ago, Misha made a gift of $2 million to create the Anatole Katok Chair in Mathematics here and together with Sergey Ferleger and Alexey Kononenko made major donations to endow the Anatole Katok Center for Dynamical Systems and Geometry. It is fitting that Yasha serves as its director. And that accordingly, even the county in which we find ourselves is called Centre county.

At the University of Maryland, the Michael and Eugenia Brin Chair in Mathematics was created by the Brins, and a match from the Maryland E-Nnovation Initiative created a second endowed chair of the same name.

The Brin Mathematics Research Center at the University of Maryland was founded less than a year ago and supports long programs, conferences & workshops, special lecture series, and summer schools. It provides ample opportunities for short-term and long-term visitors interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

Last year, the American Mathematical Society launched a new primary journal, Communications of the American Mathematical Society, dedicated to publishing the very best research and review articles across all areas of mathematics. It is completely free to authors and readers. This was made possible by quiet support from Misha for the costs of publishing this journal. In like manner, this Center supports Mathematics Research Reports, which is also free to readers and authors. Those having traveled here with support from the Center should feel obliged to submit articles to both journals!

It is a great privilege to express here much gratitude and admiration for all that Yasha and Misha have done and accomplished for mathematics, for the profession, and for our community. I do so on behalf of the American Mathematical Society, of our community, of myself, and surely of all of us here. Many thanks and congratulations!