"Fuchsian Groups" by S. Katok
Errata

p.7 l.10: Replace "Theorem 1.1.2" by "Theorem 1.2.1".
p.20 l.4: Delete an extra cos in (iii).
p.21 l.4: Replace "imaginary axis" by "positive imaginary axis".
p.21 l.8: Replace "§1.2" by "§1.3".
p.23 l.4: Replace "its" by "their".
p.24 l.7: Replace "1.2.4" by "1.2.5".
p.25 l.-11 to p.26 l.6 Replace by

Besides being a group, $\mathrm{PSL}(2,\mathbb{R})$ is also a topological space. More precisely, $\mathrm{SL}(2,\mathbb{R})$ can be identified with the subset of $\mathbb{R}^4$,

$$X = \{(a,b,c,d) \in \mathbb{R}^4 \mid ad - bc = 1\}.$$

The norm on $\mathrm{SL}(2,\mathbb{R})$ is induced from $\mathbb{R}^4$: for $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $ad - bc = 1$, we define

$$\|A\| = \sqrt{a^2 + b^2 + c^2 + d^2}, \tag{0.1}$$

and $\mathrm{SL}(2,\mathbb{R})$ is topologized with respect to the metric

$$d(A,B) = \|A - B\|. \tag{0.2}$$

Since $A \sim -A$ is an equivalence relation on $\mathrm{SL}(2,\mathbb{R})$, the factor space $\mathrm{SL}(2,\mathbb{R})/\sim = \mathrm{PSL}(2,\mathbb{R})$, is topologized with the factor topology. Exercise 2.1 shows that, in fact, $P\mathrm{SL}(2,\mathbb{R})$ is a topological group. Since by (1.3.2), orientation–reversing isometries are given by matrices in $GL(2,\mathbb{R})$ with determinant -1, the whole group of isometries Isom $(\mathcal{H})$ can be topologized using the same distance. Notice that since $\|A\| = \|-A\|$, the norm (2.1.3) is a well–defined function on $\mathrm{PSL}(2,\mathbb{R})$ while the metric (2.1.4) is not. One natural way to introduce a metric on $\mathrm{PSL}(2,\mathbb{R})$ is to represent it as a matrix group $S0_o(2,1)$, the other is through a so–called *chord metric* on the unit disc obtained from the Euclidean metric on the unit sphere via stereographic projection, but both are beyond the scope of this book.

Convergence in $\mathrm{PSL}(2,\mathbb{R})$ can be expressed in matrix language as follows. If $g_n \to g$ in $\mathrm{PSL}(2,\mathbb{R})$, this means that there exist matrices $A_n, A \in \mathrm{SL}(2,\mathbb{R})$ representing g_n and g such that $\lim_{n\to\infty} \|A_n - A\| = 0$.

p.27 l.9: Replace "metric" by "locally compact metric"
p.27 l.10: Replace "homeomorphisms" by "isometries".
p.27 l.-5-l.-3: Replace "It is clear from the definition that a group G acts properly discontinuously on X if and only if each orbit is discrete and the stabilizer of each point is finite." by "Since X is locally compact, a group G acts properly discontinuously on X if and only if each orbit has no accumulation point in X, and the order of

the stabilizer of each point is finite. The first condition, however, is equivalent to the fact that each orbit of G is discrete. For, if $g_n(x) \to s \in X$, then for any $\epsilon > 0$, $\rho(g_n(x), g_{n+1}(x)) < \epsilon$ for sufficiently large n, but since g_n is an isometry, we have $\rho(g_n^{-1}g_{n+1}(x), x) < \epsilon$, which implies that x is an accumulation point for its orbit Gx, i.e. Gx is not discrete."

p.30 l.7: In Lemma 2.2.4 replace w_0 by z_0.

p.32 l.8-l.19: In the proof of Theorem 2.2.6 replace those lines by: We use Lemma 2.2.4 to see that $\{T \in \Gamma \mid T(z) \in K\} = \{T \in \mathrm{PSL}(2,\mathbb{R}) \mid T(z) \in K\} \bigcap \Gamma$ is a finite set (it is the intersection of a compact and a discrete set), and hence Γ acts properly discontinuously. Conversely, suppose Γ acts properly discontinuously, but it is not a discrete subgroup of $\mathrm{PSL}(2,\mathbb{R})$. Then there exists a sequence $\{T_k\}$ of distinct elements of Γ such that $T_k \to \mathrm{Id}$ as $k \to \infty$. Let $s \in \mathcal{H}$ be a point not fixed by any of T_k. Then $\{T_k(s)\}$ is a sequence of points distinct from s and $T_k(s) \to s$ as $k \to \infty$. Hence every closed hyperbolic disc centered at s contains infinitely many points of the Γ-orbit of s, i.e. Γ does not act properly discontinuously, a contradiction.

p.37 l.11: Delete "$\in \Gamma$".

p.37 l.-7: In Theorem 2.4.1 delete "a cyclic group,".

p.38 l.4: Replace "2.3.5" by "2.3.2" and "a finite cyclic group, and hence abelian" by "an abelian group".

p.38 l.-7-l.-4: Replace "Since $\lambda > 1$"... upto "distinct terms" by "Since $\lambda > 1$, the sequence $g_n \circ h \circ g^{-n} \to \mathrm{Id}$".

p.42: l -12: Replace "finite cyclic and therefore elementary" by "elementary by Theorem 2.4.1".

p.49 l.5: Replace "homeomorphism" by "isometry".

p.50 l.-8: Replace "hence" by "since μ is $PSL(2,\mathbb{R})$–invariant".

p.65 l.–8: Replace "a finite cyclic group (Corollary 2.4.2)" by " an elementary group (Theorem 2.4.1)".

p.66 l.-1: Delete "limit".

p.69 l.-10: Replace "σ" by "$\rho(p, z)$".

p.70 l.17: Replace "2.3.7" by "2.2.3".

p.71 l.2: Replace" 2.2.6" by "2.2.3".

p.73 l.-4: Replave "Γ" by "$\Gamma - \{\mathrm{Id}\}$".

p.73 l.-2: Replace "$T_1^{-1}(F) \cap T^{-1}(F) \neq \varnothing$" by "$T_1^{-1}(F) \equiv T^{-1}(F)$".

p.74 l.-1: Replace "there exists a" by "any".

p.75 l.1: Replace "intersecting" by "intersects".

p.75 l.10: Delete "with $\mu(\Gamma \backslash \mathcal{H}) < \infty$".

p.87 l.-1: Replace "S" by "S_0".

p.89 l.-1: Replace "1.2.8" by "1.2.6".

p.95 l.-2: Replace "Theorem 4.3.2" by "Theorem 3.5.4".

p 105 l.-3 replace "3.2.4" by "3.3.4".

p.113 l.-3: Replace "each element" by "each non–zero element".

p.117 l.4: Replace "have" by "are integers and have".
p.119 after l.9 insert "In what follows A will be a quaternion algebra satisfying (5.2.10)".
p.119 l.-9: Replace "division" by "quaternion".
p.119 l.-5: Replace "$SL(2,\mathbb{R})$" by "$M(2,\mathbb{R})$".
p.120 l.4: Replace "$b > 1$, we have $|x_2 - x_3\sqrt{a}| < \frac{1}{2b} \leq \frac{1}{2}$" by "$|b| \geq 1$, we have $|x_2 - x_3\sqrt{a}| < \frac{1}{2|b|} \leq \frac{1}{2}$"
p.123 The end of the proof of Lemma 5.3.3 was modified according to M.Katz's suggestion.
p.128 In Cor. 5.3.10 added "over $\mathbb{Q}$".
p.142 Replace 3.6 by 3.5
p.155 l.2: In the matrix replace "a" by "α".
p.156 l.7: Replace "1.9" by "1.10".
p.157 Ex .2.4 made changes
p.158 l.8: Replace "2.10" by "2.11".
p.158 l.-14: Change "Q and T" to "Q and T (or Q and T_1)".
p.160 l.12: Replace ". So ... this" by ", which".