MASS PROGRAM AT PENN STATE

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The MASS program—Mathematics Advanced Study Semesters—was founded in 1996. It is an innovative, intensive program for select groups of up to about 25 undergraduates recruited every year from around the country and brought to Penn State's campus for one semester. This program, currently in its fourth year of operation, provides a unique and mutually reinforcing blend of learning and research activities for its participants. Learning serves as a basis and provides background for research endeavors, and research projects generate the need and desire for deeper and more extensive learning.

The major components of MASS are:

- **Three courses** designed exclusively for MASS students on topics chosen from the areas of Algebra/Number Theory, Analysis, and Geometry/Topology. Each course features three, 1–hour lectures per week, a weekly meeting conducted by a MASS Teaching Assistant, weekly homework assignments, a written midterm exam, a final project, and an oral final examination/presentation.
- **Individual student research projects** which range from theoretical mathematics research to computer implementation. Some projects are related to the core courses while others are developed independently according to the interests and abilities of the student. Research projects are supervised by the MASS director, faculty, and teaching assistants.
- **A weekly working seminar** run by the director of the MASS program, which helps to unify all the other activities.
- **The MASS Colloquium**, a weekly lecture series by distinguished mathematicians, Penn State research faculty and visitors. These lectures are very instrumental in focusing interest of the MASS participants on various research areas of mathematics both during their participation in the program and later in their selection of graduate programs.
- **Individual guidance** by the director of the program and teaching assistants. As a result of the commitment of the director, TA's, and the students, this goes far beyond the ordinary notion

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of office hours. In particular, the director works with each student to design an individual plan which covers all aspects of the MASS experience and is adapted to the student's background, interests, and talent. The director regularly meets with each student to revise this plan according to the student's progress in the program.

Participants

The participants are selected from applicants currently enrolled in US colleges or universities who are juniors or seniors in the given academic year. Students who are sophomores may be admitted in exceptional cases. All participants are expected to have demonstrated a sustained interest in mathematics and a high level of mathematical ability and to have mastered basic techniques of mathematical proof. The expected background includes a full calculus sequence, basic linear algebra, and advanced calculus or basic real analysis.

The search of the participants is nationwide. We maintained a website for the program at http://www.math.psu.edu/mass/. Participants are selected based on their academic record, recommendation letters from faculty and an essay. In the course of 4 years we had participants from University fo Nebraska, Lincoln, Mount Holyoke College, SUNY at Stony Brook, University of Missouri, Rolla, University of Texas, Austin, Rensselaer Polytechnic Institute, University of Illinois, Urbana at Champaign, University of Wisconsin at Madison, Michigan State University, Clarkson University, Arizona State University, Juniata College, University of Missouri at Kansas City, University of California at Santa Cruz, Iowa State University, SUNY at Geneseo, Sara Lawrence University, Chapman College, University of Maryland, Wayne State University, College of New Jersey, and Penn State.

Funding and Financial Arrangements

Penn State provides fellowships for out-of-state students that reduce their tuition to the in-state level. Further support comes through the National Science Foundation VIGRE grant. In particular, MASS participants whose tuition in their home institution is lower than Penn State in-state tuition receive grants for the difference. The balance of the VIGRE funds is used to further decrease out-of-pocket expenses of the participants and is distributed individually based on merit and need. Additional funding of the research component of the MASS program in the form of small individual stipends is provided through REU supplements to individual NSF grants.

MASS Courses

The following courses have been offered in the MASS program:

MASS 1996:

"Explorations in Number Theory" (George Andrews, Evan Pugh Professor of Mathematics);

"Introduction to Dynamical Systems" (Anatole Katok, Raymond N. Shibley Professor of Mathematics);

"Linear Algebra in Geometry" (Victor Nistor, Professor of Mathematics).

MASS 1997:

"Arithmetic and Geometry of the Unimodular Group" (Svetlana Katok, Professor of Mathematics);

"Real Analysis" (Nigel Higson, Professor of Mathematics);

"Explorations in Geometry" (Dmitri Burago, Assistant Professor of Mathematics, 1997 Sloan Fellow).

MASS 1998:

"Number Theory: From Fermat's Little Theorem to his Last Theorem" (Ken Ono, Assistant Professor of Mathematics);

"The Exponential Universe" (John Roe, Professor of Mathematics);

"Functions and Dynamics in One Complex Variable" (Greg Swiatek, Professor of Mathematics).

MASS 1999:

"Topics in Number Theory" (Robert Vaughan Professor of Mathematics);

"Geometric Structures, Symmetry and Elements of Lie Groups" (Anatole Katok, Raymond N. Shibley Professor of Mathematics)

"Mathematical Methods in Mechanics" (Mark Levi, Professor of Mathematics).

All courses are assisted by MASS teaching assistants who are chosen from the most accomplished Ph.D. students of the Penn State Department of Mathematics. All MASS TA's significantly exceed the requirements of our Graduate Program. There is a high level of interaction with the course instructors, the seminar leader and teaching assistants including individual tutoring.

Research

Some MASS participants have produced significant pieces of mathematical research. Here are several examples:

Mark Schmitt, a MASS-96 participant from University of Illinois, developed his MASS number theory final project into a new theorem about partitions of the terms of Fibonacci-like sequences, which he presented at the 1998 Illinois Number Theory Conference.

An Nguyen, another MASS-96 student and now a graduate student in Computer Science at Stanford, rediscovered the famous value of $\lambda = 1 + \sqrt{8}$ for the appearance of period three orbits in the logistic family $f(x, \lambda) = \lambda x(1 - x)$, and then went on to discover a previously unknown bifurcation point where the second period four orbit appears: $\lambda = 1 + \sqrt{4 + \sqrt[3]{108}}$.

James Kelley, a MASS-98 participant, studied the representation of integers by quadratic forms, a classical problem in number theory. In particular, he studied a well known problem due Irving Kaplansky: What integers are of the form $x^2 + y^2 + 7z^2$ where x, y, and z are integers. Obviously, if N is of this form, then Nk^2 is also of this form (just replace x, y, and z by kx, ky, and kz). However, the converse if not necessarily true. James proved, using the theory of elliptic curves and modular forms, that every "eligible" integer N which is not a multiple of 7 and not of this form, is square-free! His paper has been submitted for publication.

John Voight, a MASS-98 participant, now a first year graduate student at the University of California at Berkeley, wrote an extremely impressive paper on odd perfect numbers. A number is *perfect* if it equals the sum of its proper divisors. For example, 6 is perfect since it equals the sum of 1, 2 and 3, the proper divisors of 6. One of the most famous open problems in number theory is the conjecture that there are no odd perfect numbers. John proved a number of highly technical results regarding the prime factors of any alleged odd perfect number. The conditions he obtains place so many restrictions that his work may be viewed as further evidence that there are indeed no odd perfect numbers.

Credits

Students receive 15 credits for the courses and the seminar which are transferable to their home universities and a certificate from the MASS Program at Penn State. Additional recognition is provided through prizes and certificates for achievements in solving problems and for best projects. Each student is also issued a Supplement for

MASS Certificate which includes the list of MASS courses with credits, grades, final presentations, and special achievements. It also includes the descriptions of MASS courses, the list of MASS Colloquia and the description of MASS program exams. These supplements are very useful for the student's home institution equivalences and enhance the student's applications to graduate schools.

Administration

The overall supervision of the MASS program is provided by the Scientific Advisory Board which includes senior members of Penn State's mathematics faculty, and several outstanding mathematicians from other institutions.

Currently the MASS program is managed by the Acting Director chosen among senior Ph.D. students with previous experience as MASS-TAs under the supervision of Professor A. Katok, who chairs the Scientific Advisory Board. Organizational matters are attended by the Administrative Assistant. We hope to be able to hire a permanent faculty Director of the MASS program at the tenure or tenure–track level for the Fall of 2000.

Feedback

MASS students master key tools and techniques of mathematical research and they are exposed to mathematical ideas from many areas of current interest. This is an ideal preparation for a career as a research mathematician, and we are pleased to report that MASS alumni go on to graduate schools in mathematical sciences. We expect them to become active researchers in academia or in cutting-edge industrial and government research establishments. Here is a list of most of the graduate schools where the MASS alumni matriculated. In mathematics: University of Michigan, Cornell University, Penn State, Harvard University, University of Chicago, Boston University, Ohio State University, University of Wisconsin, University of Illinois at Urbana Champagne, University of Utah, University of California, Berkeley; in computer science: Stanford University. Many MASS alumni have had a choice between a number of top rated graduate schools. It is worth noticing that MASS experience not only helped them to obtain these offers, but was instrumental in helping them to make choices based on well–developed mathematical interests and not simply on general ranking of the departments.

The MASS Program boasts a number of features which make it unique among mathematics programs for undergraduates in the U.S., quite distinct from honors programs, math clubs, and summer educational or research programs. The principal difference is the comprehensive character of the program: all academic activities of the participants for a semester are specially designed and coordinated to enhance their learning and introduce them to research in mathematics. This produces a quantum leap effect: the achievement and enthusiasm of MASS students increases much more dramatically than if they had been exposed to similar amounts of material over a longer time in more ordinary circumstances. A key feature of the MASS experience is the intense and productive interaction that takes place among the students. The environment is designed to encourage such interaction: a classroom is dedicated to MASS and furnished so as to serve as a lounge and a computer lab outside of class times. The students live together in a contiguous block of dorm rooms, they eat together, and they pursue various social activities together. The effect of such conditions is dramatic: the students find themselves members of a cohesive group of like–minded people sharing a special formative experience. They quickly bond, and often remain friends after the program is over. They study together, attack problems together, debug programs together, collaborate on research projects and, most importantly, talk about mathematics all the time. Of course, this is exactly how "mature" mathematicians operate in their professional life. A necessary condition for this environment is the gathering of a "critical mass" of dedicated and talented math students, which is one of the chief accomplishments of MASS.

Proof that the MASS program is working as hoped, is confirmed by a variety of observations gained from the students' assessments.

Suzanne Lynch, who participated in MASS-96 and is now a graduate student at Cornell, wrote this in an unsolicited letter:

The MASS program has been the best semester of my life. I was immersed in an environment of bright motivated students and professors and challenged as never before. I was pushed by instructors, fellow students and something deep inside myself to work and learn about mathematics, and my place in the mathematical world. I loved my time there, and never wanted to leave. I believe the MASS program helped to prepare me for the rigors of graduate school, academically and emotionally. . . . The MASS program has been very instrumental in opening grad school doors to me, and giving me the courage to walk through them.

The impact of MASS on the students is reflected in comments from their post-program questionnaires, which were uniformly laudatory.

I found the MASS program to be a great cutting edge learning experience. To me it was a preview of the things to come, and although I could not always keep up with all my class work I still enjoyed it a lot. ... I had no definite career goal prior to MASS. Now I have decided to attend graduate school and I'm positive that MASS has influenced my decision. Also, I feel, it made me more confident in my learning ability. – Daniel Genin, MASS-96 and MASS-97, now a graduate student at Penn State.

The MASS Program has taught me how to do, live and breathe MATH. I've come out of this program not a brainy robot with courses under my belt, but a deep thinker and philosophical person toward life. ... the MASS Program has been the best education in my life, and I recommend anyone who seeks professionalism to take this program at LEAST ONCE!!! – Chris Staskewicz, MASS-97, now a graduate student at University of Utah

The only word that I can use to describe the MASS Program is intense. ... I know that the MASS Program has helped me focus my mathematical interests. After completing this Program, I feel that I can accomplish anything. I feel like I have so much more mathematical ability or at least the confidence that I can tackle the many challenging problems that are ahead. – Colleen Kilker, MASS-97, now a graduate student at University of Illinois at Urbana-Champaign

I would not be in the strong position I am now in my application to graduate school without MASS. My professors back at home were quite amazed with my GRE score, and that had a lot to do with the environment and the challenge provided by the MASS program. It also helped me to figure out that my proclivities lie in number theory, something that would have taken me much longer to figure out absent the intensive approach taken in each of the three courses. I made a lot of friends I will miss dearly. I feel more confident generally in my mathematical abilities. John Voight, MASS-98, now a graduate student at UC Berkeley

The MASS program is a great thing. When I return to my home school, I will certainly recommend it, and tell the people there about my positive math experience at PSU. I have never been pushed this hard, or learned this much in a single semester. The mathematics was fantastic, I am very excited about mathematics and my background was greatly *expanded and my perspective enlarged.* Matthew Boylan, MASS-98, now a graduate student at Penn State and a VIGRE fellow.

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