REU SUMMER PROGRAM AT PENN STATE

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The Penn State Summer REU program started in 1999 as an extension of the MASS (Mathematics Advanced Study Semesters) Program. It is a 6-week program for undergraduate students from United States colleges and universities. Participants are selected based on their academic record, recommendation letters from faculty and an essay. Prerequisites for the participants include an equivalent of at least two years of college mathematics for mathematics/science majors. Normally a straight A record in those courses is required. Support is provided by the National Science Foundation VIGRE grant.

The goal of the program is to expose the participants to different areas of mathematics and let them experience a combination of advanced learning and research initiation. Any mathematical research usually includes three components: study of the subject, solving of a problem, and presentation of the result. We attempted to include all three components into our program. Thus in addition to the traditional for REU's individual or small group research projects supervised by faculty mentors, the program includes two short courses, a weekly seminar, and a three-day "MASS-FEST" conference.

All elements of the program are specially designed for REU participants. A crucial feature of the program is a high level of interaction between the participants and the REU faculty. A special effort is made to coordinate and mutually reinforce content of the courses and other activities so that various mathematical concepts and methods appear in different settings.

Participants

The search of the participants was nationwide. We maintained a web-site for the program at:
http://www.math.psu.edu/mass/Reu99/reu.html/.

The information about the program was also included into the NSF REU list. We received 52 applications, made 19 offers out of which 8 were declined and 11 accepted. The participants came from SUNY

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College at Geneseo, University of Notre Dame, Penn State, University of Oregon, Chapman University, University of California, Berkeley, University of Nebraska at Lincoln. They included 1 freshman, 2 sophomores, 7 juniors, 1 senior; 2 females and 9 males.

Funding

The main source of support was the NSF VIGRE grant ($35,200). Two Penn State faculty members have contributed their individual REU NSF grant funds ($9,500) to the budget; they provided research projects for 3 students and served as their faculty mentors. Each participant received a $1,800 stipend, travel reimbursement up to $500, and the reimbursement for room and board in the amount of $900. The budget for the MASS-FEST conference was $5,500 which provided hotel accommodations, per diem, and a modest travel allowance for MASS alumni.

REU Courses

The two two–week short courses given in the REU-99 were "Symmetry and Representations of Finite Groups" (June 21 – July 2) instructed by Adrian Ocneanu, (Penn State, Professor of Mathematics), and "The Theory of Partitions" (July 7 – July 16) instructed by George Andrews, (Penn State, Evan Pugh Professor of Mathematics). The weekly seminar was instructed by the REU coordinator Misha Guysinsky, (Tufts University, Assistant Professor of Mathematics). The following are short descriptions of the courses.

"Symmetry and Representations of Finite Groups" This course covered the basic ideas of the theory of representations of finite groups and the structure of their tensor products. The material that is usually part of a graduate course was approached concretely by using familiar examples such as the cyclic and dihedral groups. The presentation was made in a manner that encouraged the students to build the whole theory by themselves as answers to a series of simple questions. Topics covered: Real and complex vector spaces; Direct sums and tensor products of vector spaces; The structure of groups; Groups given by generators and reflections; Geometrical interpretation of the trace, determinant, and characteristic polynomial; Exponentials of matrices; Multiplicative defect for products of exponentially non-commuting matrices; Rotations as imaginary exponentials; Representations of groups by linear transformations; Change of base and irreducible representations; Inequivalent representations; The structure of finite Abelian groups and their irreducible representation; Tensor products of representations and their decomposition into irreducible components; Characters of representations; The McKay graph of tensoring with a distinguished representation of a group; The Peter-Weyl Theorem on the orthogonality and completeness of matrix entries irreducible representations; The order of the group as the sum of squares of the dimensions of its irreducible representations.

"The Theory of Partitions" This course covered the basic topic in additive number theory, the theory of partitions. This topic begins with the fundamental question: "How many ways can you write a given integer as sums of positive integers?" In trying to answer this question the students were led to a variety of intriguing discoveries including some of those found by India's great, enigmatic genius, Srinivasa Ramanujan. The Omega Package (implemented in Mathematica) has been introduced in order to use the power of modern computer algebra to investigate partitions. Topics covered: I. Partitions (Definitions; Infinite product generating functions; Ferrers graphs; The Pentagonal Number Theorem and its applications). II. q-Series (The idea of q-analogs; The q-Binomial Series; Heine's transformation and its iterates; Applications of Heine's theorem to partitions). III. MacMahon's Partitions Analysis (Basic definitions; Basic transformation rules; Triangles with integer sides; Partitions with non-negative higher order differences; Evaluation of binomial sums).

Research Projects

An advantage of our program is that many participants return to Penn State in the fall for the MASS program. This allowed us to offer the students some research projects which required more than 6 weeks for completion. The research projects in various areas of mathematics were provided by interested faculty of our department. Some projects were directly related to topics included in the short courses. We also tried to involve students in more than one project. The projects included:

1. "Matrices with Non-conjugate Centralizers" (1 student; faculty mentor S. Katok, Professor of Mathematics)
   The student who worked on this project found two matrices in $SL(3,\mathbb{Z})$ which have the same eigenvalues, but are not conjugate in $GL(3,\mathbb{Z})$, and therefore their centralizers are not conjugate. The existence of such matrices was guaranteed by the theory based on the connection between classes of conjugate matrices and ideal classes of rings of integers in algebraic number fields. The example

was obtained from a totally real cubic field of discriminant 1957 with the use of a package of programs called PARI. This provides an example of interesting new phenomena in the theory of actions of higher rank abelian groups.

2. "A Geometric Spectral Theory for n–tuples of Self-Adjoint Matrices and their Generalizations" (2 students; faculty mentor J. Anderson, Professor of Mathematics)
   The students studied the general facts about the spectral theory of self-adjoint operators, spectral scale theory and made several explicit calculations of spectral scale including a case with multiplication operators defined on an infinite dimensional space.
3. "Reflection Groups and Their Symmetries" (1 student; faculty mentor A. Ocneanu, Professor of Mathematics)
   The problem was to find a canonical construction that allows to find a unimodular cover for any Dynkin diagram. The student had to study the theory of the reflection groups and their classification. He suggested an idea how the construction could be carried out.
4. "Simplices with Only One Integer Point" (2 students; faculty mentor A. Borisov, Assistant Professor of Mathematics)
   The students found an effective procedure that allowed to find all classes of simplices with vertices that have only integer coordinates and only one point with integer coordinates inside. Using computers they found all classes for the dimensions 3 and 4.
5. "The Growth Rate of the Number of Generalized Diagonals for a Polygonal Billiard" (2 students; faculty mentor A. Katok, Raymond N. Shibley Professor of Mathematics)
   The students had to study the number of different codings for polygonal billiards. For any trajectory there is an isometry which corresponds to unfolding of the trajectory after n-reflections. Students wrote a program that allows to find all possible codings and corresponding isometries for any convex rational polygon. They used this program to study a regular octagon and found that the isometries that come from codings belong to a certain surface.
6. "MacMahon's Binomial Coefficient Powers" (1 student; faculty mentor G. Andrews, Evan Pugh Professor of Mathematics)
   The problem was to find for what non-negative integer numbers a_1,a_2,. . .,a_n the following product

$$\left(\frac{N-n+1}{1}\right)^{a_1} \cdot \left(\frac{N-n+2}{2}\right)^{a_2} \cdots \left(\frac{N}{n}\right)^{a_n}$$

   is an integer number for any integer $N \geq n$. The student used ideas developed by MacMahon in the beginning of the century and also Omega-package developed last year. He solved the problem for $n \leq 7$. He also indicated the problems and suggested methods for their solutions for $n > 7$.

7. "Triangle Chain Partitions" (1 student; faculty mentor G. Andrews, Evan Pugh Professor of Mathematics)
   The student solved a problem from the partition theory about the number of non-congruent chains of triangles with integer sides. He used Omega-package in order to make intermediate calculations. He also tried to solve a similar problem for tetrahedrons and obtained partial results.
8. "New Congruences for the Partition Function" (1 student; faculty mentor K. Ono, Assistant Professor of Mathematics). This project started before the REU program began. Using the theory of Hecke operators for modular forms of half-integral weight, the student found an algorithm for primes $13 \leq m \leq 31$ which reveals $70,266$ new congruences of the form $p(An + B) \equiv 0 \pmod{m}$, where $p(n)$ denote the number of unrestricted partitions of a non-negative integer n. As a simple example, she proved that $p(3828498973n+1217716) = 0 \pmod{13}$ for every integer n. The first three congruences were found in 1919 by Ramanujan, and after that finding new ones was considered as a very difficult problem. The paper of this student has been accepted for publication by the Ramanujan Journal.

MASS-FEST Conference

At the end of the program we had a 3-day MASS-FEST conference. In addition to REU participants and Penn State faculty and graduate students, several MASS alumni, who are now in different Graduate programs came to Penn State for the MASS-FEST. It featured talks by Professors A. Katok ("Variational Calculus and Closed Geodesics"), A. Kouchnirenko ("Newton Polygons"), K. Ono ("Ramanujan's Congruences and More"), J. Roe ("Are Infinitely Many Dimensions Enough?"), R. Vaughan ("Means of Exponential Sums"), participants of the REU program and MASS alumni. For the REU participants this was a very valuable experience. It allowed them to interact with other young mathematicians, more advanced in their mathematical careers than themselves, and see such mathematicians in action. It also gave them an insight into the MASS Program which some REU participants will attend in the fall. Among 11 REU-99 participants 2 were MASS alumni and 5 will attend the MASS program in the Fall of 1999.

Social Events

The REU participants were housed in the same block of rooms in one of the Residence Halls. The first two weeks we organized several social events in order to help them to get better acquainted with each other and Penn State faculty. Social events included pizza party, ice cream party, trips to a lake and a skating ring. During the MASS-FEST we organized a dinner, a party, and a trip to Penn Caves. At the end of the program we presented 4 students with the following awards:

"Excellence in Project Presentation" (2 students),
"Most Industrious Participant" (1 student),
"Most Outstanding Research Results" (1 student).

Student Comments

I enjoyed the 6 week format because it gave me an opportunity to see some complex and in depth math during the summer and still left time to pursue other interests. The conference at the end was great. I got a feeling for presenting mathematical research in front of an audience.

The program was very well administered and it was clear that the professors and administers alike really cared about the students and that we learned.

Overall this REU was a great and useful mathematical experience!

I believe the program was organized and carried out exceptionally well. The courses and lectures were absolutely excellent. Always try to find professors like them.

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